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First Semester M.Tech. Degree Examination, June/July 2023
Mathematical Foundation of Computer Science

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing ONE full question from each module.***
2. ***t-distribution table and χ^2 – distribution table allowed.***

Module-1

1 a. (i) Define the terms subspace and linear span of a set.
(ii) Is S and T are subspaces of the vector space V(F), then $S \cap T$ is a subspace of V(F). **(10 Marks)**

b. (i) Define the terms linearly independent of a set and coordinates.
(ii) Let $S = \{v_1, v_2, v_3, v_4\}$ be a basis for $\mathbb{R}^4$.
Where $v_1 = (1, 0, 0, 0)$, $v_2 = (2, 0, 1, 0)$, $v_3 = (0, 1, 2, -1)$ and $v_4 = (0, 1, -1, 0)$.
If v = (1, 2, -6, 2). Compute coordinate vector of v. **(10 Marks)**

OR

2 a. (i) Define the terms basis and dimension.
(ii) Find a basis and the dimension of the subspace,

$$W = \left\{ \begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} : a, b, c, d \text{ are real} \right\}$$

(10 Marks)

b. (i) Define a linear transformation.
(ii) Find the matrix of linear transformation.
$T : V_2(R) \rightarrow V_3(R)$ defined by
T(x, y) = (x+y, x, 3x-y) relative to the basis
$B_1 = \{(1, 1), (3, 1)\}$ and $B_2 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ **(10 Marks)**

Module-2

3 a. (i) Define the terms inner product and orthogonal sets.
(ii) If $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, find the orthogonal projection of y onto u and the orthogonal set. Also write y as the sum of two orthogonal vectors. **(08 Marks)**

b. Find the QR factorization for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. **(12 Marks)**

OR

4 a. Using Gram-Schmidt process, find an orthogonal basis for W = span{x_1, x_2, x_3} in $\mathbb{R}^3$ if $x_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$, $x_2 = \begin{bmatrix}8\\1\\6\end{bmatrix}$ and $x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$. **(10 Marks)**

b. Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points (2, 1), (5, 2), (7, 3) and (8, 3). **(10 Marks)**

Module-3

5 a. Diagonalize for the following matrix :

$$A = \begin{bmatrix}6 & -2 & -1\\-2 & 6 & -1\\-1 & -1 & 5\end{bmatrix}.$$ **(12 Marks)**

b. (i) Define constrained optimization.

(ii) Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$.

Subject to the constraint $X^T X = 1$. **(08 Marks)**

OR

6 Find the singular value decomposition of $A = \begin{bmatrix}4 & 11 & 14\\8 & 7 & -2\end{bmatrix}$. **(20 Marks)**

Module-4

7 a. Fit a parabola of the form $y = a + bx + cx^2$ to the following data using method of least squares :

x	0	1	2	3	4	5
y	1	3	7	13	21	31

Also estimate y at x = 6 **(10 Marks)**

b. Find the correlation coefficient and the regression lines of y on x and x on y for the following data :

x	1	2	3	4	5
y	2	5	3	8	7

(10 Marks)

OR

8 a. Fit a non-linear curve of the form $y = ae^{bx}$ to the following data from the method least squares.

x	1	5	7	9	12
y	10	15	12	15	21

(10 Marks)

b. If θ is the acute angle between the two regression lines, then show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$. Explain the significance of the formula when r = 0 and r = ±1.

(10 Marks)

Module-5

9 a. (i) Is the function defined as follows a density function?

$$f(x)=\begin{cases} e^{-x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

(ii) If so, determine the probability that the variate having this density will fall in the interval (1, 2)?

(iii) Also find the cumulative probability function F(2)?

(iv) Find $E(X^2)$ and $E(4X^2)$. **(10 Marks)**

b. A set of five similar coins is tossed 320 times and the result is,

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution at 5% level of significance. **(10 Marks)**

OR

10 a. A random variable X has the following probability mass function :

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find :

(i) The value of K

(ii) $P(X<6), P(X \ge 6)$

(iii) $P(0<X<5)$

(iv) F(3) **(10 Marks)**

b. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results :

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether you can discriminate between the two horses. **(10 Marks)**

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